Chapter 4 Problem Set – Rational Functions, Equations and Inequalities.

4.1 Introduction to Rational Functions and Asymptotes. #1, 2, 3 (5.2 in textbook)

1. Without using graphing technology, match each equation with its corresponding graph. Explain your reasoning.

a) $y = \dfrac{-1}{x - 3}$

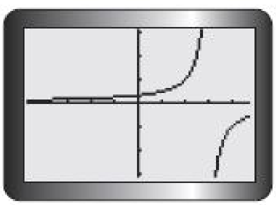
b) $y = \dfrac{x^2 - 9}{x - 3}$

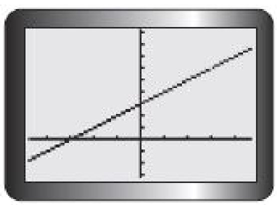
c) $y = \dfrac{1}{(x + 3)^2}$

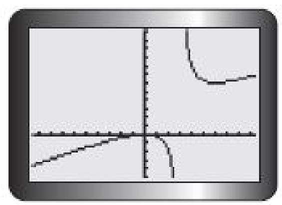
d) $y = \dfrac{x}{(x - 1)(x + 3)}$

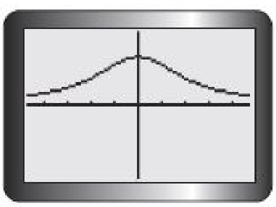
e) $y = \dfrac{1}{x^2 + 5}$

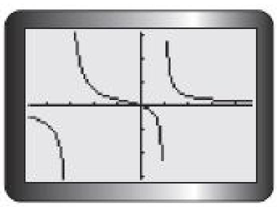
f) $y = \dfrac{x^2}{x - 3}$

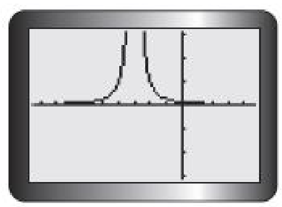
A


C


E


B


D


F


2. For each function, determine the equations of any vertical asymptotes, the locations of any holes, and the existence of any horizontal or oblique asymptotes.

a) $y = \dfrac{x}{x + 4}$

b) $y = \dfrac{1}{2x + 3}$

c) $y = \dfrac{2x + 5}{x - 6}$

d) $y = \dfrac{x^2 - 9}{x + 3}$

e) $y = \dfrac{1}{(x + 3)(x - 5)}$

f) $y = \dfrac{-x}{x + 1}$

g) $y = \dfrac{3x - 6}{x - 2}$

h) $y = \dfrac{-4x + 1}{2x - 5}$

i) $y = \dfrac{8x}{4x + 1}$

j) $y = \dfrac{x + 4}{x^2 - 16}$

k) $y = \dfrac{x}{5x - 3}$

l) $y = \dfrac{-3x + 1}{2x - 8}$

3. Write an equation for a rational function with the properties as given.

a) a hole at $x = 1$

b) a vertical asymptote anywhere and a horizontal asymptote along the x-axis

c) a hole at $x = -2$ and a vertical asymptote at $x = 1$

d) a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = 2$

e) an oblique asymptote, but no vertical asymptote

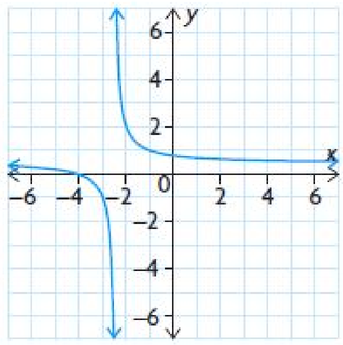
4.2 Graphs of Rational Functions. #1, 2, 4, 5, 6, 9, 10 (5.3 in textbook)

1. Match each function with its graph.

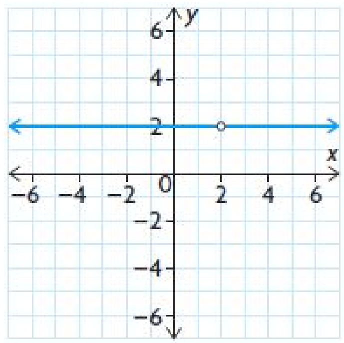
a) $h(x) = \dfrac{x+4}{2x+5}$ c) $f(x) = \dfrac{3}{x-1}$

b) $m(x) = \dfrac{2x-4}{x-2}$ d) $g(x) = \dfrac{2x-3}{x+2}$

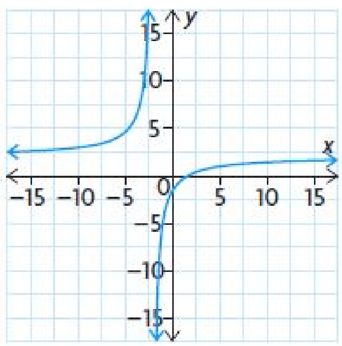
A



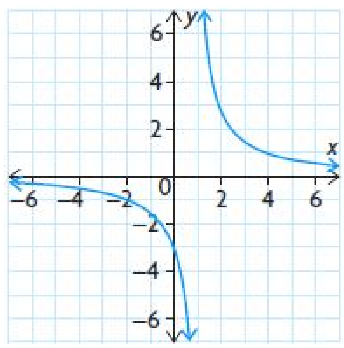
C



B



D



2. Consider the function $f(x) = \dfrac{3}{x-2}$.

a) State the equation of the vertical asymptote.

b) Use a table of values to determine the behaviour(s) of the function near its vertical asymptote.

c) State the equation of the horizontal asymptote.

d) Use a table of values to determine the end behaviours of the function near its horizontal asymptote.

e) Determine the domain and range.

f) Determine the positive and negative intervals.

g) Sketch the graph.

4. State the equation of the vertical asymptote of each function. Then choose a strategy to determine how the graph of the function approaches its vertical asymptote.

a) $y = \dfrac{2}{x+3}$ c) $y = \dfrac{2x+1}{2x-1}$

b) $y = \dfrac{x-1}{x-5}$ d) $y = \dfrac{3x+9}{4x+1}$

5. For each function, determine the domain, intercepts, asymptotes, and positive/negative intervals. Use these characteristics to sketch the graph of the function. Then describe where the function is increasing or decreasing.
K

a) $f(x) = \frac{3}{x + 5}$

b) $f(x) = \frac{10}{2x - 5}$

c) $f(x) = \frac{x + 5}{4x - 1}$

d) $f(x) = \frac{x + 2}{5(x + 2)}$

6. Read each set of conditions. State the equation of a rational function of the form $f(x) = \frac{ax + b}{cx + d}$ that meets these conditions, and sketch the graph.

a) vertical asymptote at $x = -2$, horizontal asymptote at $y = 0$; negative when $x \in (-\infty, -2)$, positive when $x \in (-2, \infty)$; always decreasing

b) vertical asymptote at $x = -2$, horizontal asymptote at $y = 1$; x-intercept $= 0$, y-intercept $= 0$; positive when $x \in (-\infty, -2)$ or $(0, \infty)$, negative when $x \in (-2, 0)$

c) hole at $x = 3$; no vertical asymptotes; y-intercept $= (0, 0.5)$

d) vertical asymptotes at $x = -2$ and $x = 6$, horizontal asymptote at $y = 0$; positive when $x \in (-\infty, -2)$ or $(6, \infty)$, negative when $x \in (-2, 6)$; increasing when $x \in (-\infty, 2)$, decreasing when $x \in (2, \infty)$

9. The function $I(t) = \frac{15t + 25}{t}$ gives the value of an investment, in thousands of dollars, over t years.

a) What is the value of the investment after 2 years?

b) What is the value of the investment after 1 year?

c) What is the value of the investment after 6 months?

d) There is an asymptote on the graph of the function at $t = 0$. Does this make sense? Explain why or why not.

e) Choose a very small value of t (a value near zero). Calculate the value of the investment at this time. Do you think that the function is accurate at this time? Why or why not?

f) As time passes, what will the value of the investment approach?

10. An amount of chlorine is added to a swimming pool that contains pure water. The concentration of chlorine, c, in the pool at t hours is given by $c(t) = \frac{2t}{2 + t^2}$, where c is measured in milligrams per litre. What happens to the concentration of chlorine in the pool during the 24 h period after the chlorine is added?
A

4.3 Solving Rational Equations. #2, 5, 6, 7, 9, 12, 13 (5.4 in textbook)

2. Solve each equation algebraically. Then verify your solution using graphing technology.

a) $\frac{x+3}{x-1}=0$

b) $\frac{x+3}{x-1}=2$

c) $\frac{x+3}{x-1}=2x+1$

d) $\frac{3}{3x+2}=\frac{6}{5x}$

5. Solve each equation algebraically.

d) $\frac{2}{x+1}+\frac{1}{x+1}=3$

e) $\frac{2}{2x+1}=\frac{5}{4-x}$

f) $\frac{5}{x-2}=\frac{4}{x+3}$

6. Solve each equation algebraically.

d) $x+\frac{x}{x-2}=0$

e) $\frac{1}{x+2}+\frac{24}{x+3}=13$

f) $\frac{-2}{x-1}=\frac{x-8}{x+1}$

7. Solve each equation using graphing technology. Round your answers to two decimal places, if necessary.

d) $\frac{1}{x}-\frac{1}{45}=\frac{1}{2x-3}$

e) $\frac{2x+3}{3x-1}=\frac{x+2}{4}$

f) $\frac{1}{x}=\frac{2}{x}+1+\frac{1}{1-x}$

9. The Greek mathematician Pythagoras is credited with the discovery of the Golden Rectangle. This is considered to be the rectangle with the dimensions that are the most visually appealing. In a Golden Rectangle, the length and width are related by the proportion $\frac{l}{w}=\frac{w}{l-w}$. A billboard with a length of 15 m is going to be built. What must its width be to form a Golden Rectangle?

12. **A** Polluted water flows into a pond. The concentration of pollutant, c, in the pond at time t minutes is modelled by the equation $c(t)=9-90\,000\left(\frac{1}{10\,000+3t}\right)$, where c is measured in kilograms per cubic metre.

a) When will the concentration of pollutant in the pond reach 6 kg/m^3?

b) What will happen to the concentration of pollutant over time?

13. **T** Three employees work at a shipping warehouse. Tom can fill an order in s minutes. Paco can fill an order in $s-2$ minutes. Carl can fill an order in $s+1$ minutes. When Tom and Paco work together, they take about 1 minute and 20 seconds to fill an order. When Paco and Carl work together, they take about 1 minute and 30 seconds to fill an order.

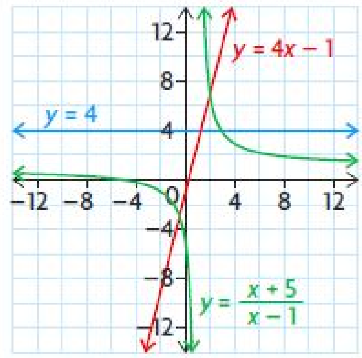
a) How long does each person take to fill an order?

b) How long would all three of them, working together, take to fill an order?

4.4 Solving Rational Inequalities. #1, 3, 4, 5, 6, 9, 11 (5.5 in textbook)

1. Use the graph shown to determine the solution set for each of the following inequalities.

a) $\frac{x+5}{x-1} < 4$ b) $4x - 1 > \frac{x+5}{x-1}$



3. a) Show that the inequality $x + 2 > \frac{15}{x}$ is equivalent to the inequality $\frac{(x+5)(x-3)}{x} > 0$.

b) Use a table to determine the positive/negative intervals for $f(x) = \frac{(x+5)(x-3)}{x}$.

c) State the solution to the inequality using both set notation and interval notation.

4. Use algebra to find the solution set for each inequality. Verify your answer using graphing technology.

d) $\frac{7}{x-3} \geq \frac{2}{x+4}$

e) $\frac{-6}{x+1} > \frac{1}{x}$

f) $\frac{-5}{x-4} < \frac{3}{x+1}$

5. Use algebra to obtain a factorable expression from each inequality, if necessary. Then use a table to determine interval(s) in which the inequality is true.

d) $t - 1 < \frac{30}{5t}$

e) $\frac{2t-10}{t} > t + 5$

f) $\frac{-t}{4t-1} \geq \frac{2}{t-9}$

6. Use graphing technology to solve each inequality.

d) $\frac{x}{x+9} \geq \frac{1}{x+1}$

e) $\frac{x-8}{x} > 3 - x$

f) $\frac{x^2-16}{(x-1)^2} \geq 0$

9. The equation $f(t) = \frac{5t}{t^2 + 3t + 2}$ models the bacteria count, in thousands, for a sample of tap water that is left to sit over time, t, in days. The equation $g(t) = \frac{15t}{t^2 + 9}$ models the bacteria count, in thousands, for a sample of pond water that is also left to sit over several days. In both models, $t > 0$. Will the bacteria count for the tap water sample ever exceed the bacteria count for the pond water? Justify your answer.

A

11. An economist for a sporting goods company estimates the revenue and cost functions for the production of a new snowboard. These functions are $R(x) = -x^2 + 10x$ and $C(x) = 4x + 5$, respectively, where x is the number of snowboards produced, in thousands. The average profit is defined by the function $AP(x) = \frac{P(x)}{x}$, where $P(x)$ is the profit function. Determine the production levels that make $AP(x) > 0$.

T